# The Effect of Primordial Non-Gaussianities on the Seeds of Super-Massive Black Holes

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The origin of the seeds which develop into the observed super-massive black holes at high redshifts may be hard to interpret in the context of the standard $\Lambda CDM$ of early universe cosmology based on Gaussian primordial perturbations. Here we consider the modification of the halo mass function obtained by introducing skewness and kurtosis of the primordial fluctuations. We show that such primordial non-Gaussianities constrained by the current observational bounds on the nonlinearity parameters of $f_{NL}$ and $g_{NL}$ are not effective at greatly increasing the number density of seeds which could develop into super-massive black holes at high redshifts. This is to be contrasted with the role which cosmic string loops could play in seeding super-massive black holes.

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## I. INTRODUCTION

Super-massive black holes (SMBHs) with masses exceeding $10^6 M_\odot$ ($M_\odot$ denotes the solar mass) have recently attracted a lot of attention [1–13]. It is now believed that each galaxy contains at least one super-massive black hole which forms from the accretion of gas about massive seed objects. The origin of the seeds which cause the formation of SMBHs is still somewhat of a mystery [14, 15]. According to the standard paradigm of early universe in which the primordial cosmological fluctuations are approximately Gaussian and have an almost scale invariant spectrum, nonlinearities form only at fairly late times and there may not be enough time to produce the nonlinear massive seeds which seed SMBHs of mass greater or equal to $10^9 M_\odot$ at redshifts of 6 or higher (of which more than 40 candidates are now known [5, 6]). There are three types of candidate seeds for SMBHs, namely Population III stars with masses in the range between $10^2 M_\odot$ and $10^3 M_\odot$, dense matter clouds with masses between $10^3 M_\odot$ and $10^6 M_\odot$, and compact objects of mass between $10^2 M_\odot$ and $10^4 M_\odot$ formed by the collision of old stellar clusters.

Recently [16] it has been shown that cosmic string loops which result from a scaling solution of strings formed during a phase transition in the very early universe lead to an additional source of compact seeds. The number density of string-induced seeds dominates at high redshifts and can help trigger the formation of the observed super-massive black holes. Cosmic string loops form a special type of non-Gaussian density fluctuation field. Non-Gaussianities in the primordial density perturbation field are an inevitable consequence of the nonlinearities of the Einstein field equations. However, in minimal models of matter and in the context of an inflationary origin of the density fluctuations such non-Gaussianities have a very small amplitude. Larger amplitudes can be obtained in non-minimal models of inflation [17] and in some alternatives to inflation such as the "Ekpyrotic" scenario [18] and in the "matter bounce" [19]. In such models, the non-Gaussianities are usually parametrized in terms of the skewness and kurtosis which characterize the deviations from Gaussianity in the three and four point functions. Skewness and kurtosis are a good characteristic of non-Gaussianities emerging from a distortion of originally Gaussian fluctuations. In this paper we ask whether such non-Gaussianities can play a similar role as cosmic string loops in the triggering of the formation of nonlinear seeds for SMBHs at high redshifts.

Primordial perturbations which originate as quantum fluctuations of a scalar field and which seed cosmic microwave background (CMB) fluctuations and structure formation of the Universe lead to non-Gaussianities which are well characterized by skewness and kurtosis (in contrast to string loop-induced fluctuations which are intrinsically non-Gaussian and hence not well characterized by the three and four point functions only. Nevertheless, the conventional non-Gaussianities resulting from a deformation of an initially Gaussian process can yield information about the early stages of the evolution of our Universe. In particular, they can allow discrimination between various models of inflation and their alternatives.

In general, the three and four point functions are characterized by an amplitude and a shape. Here, we will focus on local type non-Gaussian perturbations which can be written in the following form [20, 21]

$$\zeta = \zeta_G + \frac{3}{5} f_{NL}(\zeta_G^2 - \langle\zeta_G^2\rangle) + \frac{9}{5} g_{NL}\zeta_G^3. \tag{1}$$

where $\zeta$ is the primordial curvature fluctuation variable, $\zeta_G$ is the Gaussian part and $f_{NL}$ and $g_{NL}$ are nonlinearity parameters. These nonlinearity parameters $f_{NL}$ and $g_{NL}$ lead to non zero values of the skewness and kurtosis of the primordial non-Gaussianities in the Probability Density Function (PDF), respectively [22–25]. Current observations constrain the magnitude of the parameters $f_{NL}$ and $g_{NL}$ [26] to be $f_{NL}^{equil} = -16 \pm 79$ and $g_{NL}^{local} = (-9 \pm 7.7) \times 10^4$ (both at 68% confidence level). Here, the superscripts stand for the shape, "equilateral" in the first case and "local" in the second.

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In this paper we take into acount the modification of the halo mass function in the presence of skewness and kurtosis of local type (following the method of [22, 27]) and determine the effects of these primordial non-Gaussianities on the mass of the dark matter halos at different redshifts (up to $z = 20$) compared with the situation in a purely Gaussian model. We demonstrate that the constraints which come from the current observational data on the magnitude of the nonlinearity parameters are already strong enough to prevent the non-Gaussianities from eliminating the difficulties that the standard $\Lambda CDM$ of early universe cosmology may face in terms of explaining the origin of the observed super-massive black holes at high redshifts.

The paper is organized as follows. First we start with a short review of the halo mass function in the $\Lambda CDM$ model. In Section III we study the modifications of the halo mass function resulting from introducing non-vanishing skewness and kurtosis of the primordial density perturbations. We evaluate the result for skewness and kurtosis of equilateral shape. We show that this type of primordial non-Gaussianities obeying the current observational bounds on the nonlinearity parameters $f_{NL}$ and $g_{NL}$ is not effective at greatly enhancing the number density of nonlinear seeds for SMBHs at high redshifts. We conclude with a discussion section.

## II. HALO MASS FUNCTION IN THE $\Lambda CDM$ MODEL

The linear relative matter density fluctuation can be written in terms of the primordial curvature fluctuations $\zeta(k)$ on uniform energy density hypersurfaces by [22, 28]

$$\delta(\vec{k}, z) = \frac{2k^2}{5\Omega_{m0} H_0^2} T(k) D(z) \zeta(\vec{k}) , \tag{2}$$

where $\vec{k}$ denotes comoving wavenumber, $z$ is the cosmological redshift, $\Omega_{m0}$ is the present density parameter, $H_0$ is the Hubble constant, $D(z)$ is a linear growth function and $T(k)$ is a transfer function. Hence, the linear matter power spectrum $P_\delta(k)$ is given by the curvature power spectrum $P_\zeta$ via

$$\begin{aligned} \langle \delta(\vec{k}_1, z) \delta(\vec{k}_2, z) \rangle &= (2\pi^3) P_\delta(k_1, z) \delta^3(\vec{k}_1 + \vec{k}_2) \\ P_\delta(k_1, z) &= \mathcal{A}(k_1, z)^2 P_\zeta(k_1) , \end{aligned} \tag{3}$$

where

$$\mathcal{A}(k, z) = \frac{2k^2}{5\Omega_{m0} H_0^2} T(k) D(z) . \tag{4}$$

The smothed density fluctuation on a given length, $R$, is defined by

$$\delta_R = \int \frac{d^3\vec{k}}{(2\pi)^3} W_R(k) \delta(\vec{k}, z) , \tag{5}$$

with $W_R(k)$ being the Fourier transform of a window function

$$W_R(k) = \frac{3\ (\sin kR - kR \cos kR)}{(kR)^3} . \tag{6}$$

The variance in mass on a momentum scale $k$ with a top-hat filter with a radius that encloses the mass is (for an infinite total spatial volume) given by

$$\begin{aligned} \sigma_R^2 &= \int \frac{dk}{k} W_R^2(k)\ \mathcal{A}(k, z)^2 \mathcal{P}_\zeta(k) \\ &= \int \frac{dk}{k} W_R^2(k)\ \mathcal{P}_\delta(k) , \end{aligned} \tag{7}$$

where the dimensionless power spectra $\mathcal{P}$ are related to the dimensional ones $P$ via

$$\begin{aligned} \mathcal{P}_\zeta(k) &= \frac{k^3}{2\pi^2} P_\zeta(k) = A_s(\frac{k}{k_s})^{n_s - 1} \\ \mathcal{P}_\delta(k) &= \frac{k^3}{2\pi^2} P_\delta(k) . \end{aligned} \tag{8}$$

The halo mass function based on the Press-Schechter theory [29] is given by [30]

$$\begin{aligned} \frac{dn}{dM}(M, z) dM &= -dM \frac{2\rho}{M} \frac{d}{dM} \int_{\delta_c/\sigma_R}^{\infty} d\nu F(\nu) \\ &= -dM \sqrt{\frac{2}{\pi}} \frac{\rho}{M} \exp[-\frac{\nu_c^2}{2}]\ \nu_c\ \frac{d \log \sigma_R}{dM} , \end{aligned} \tag{9}$$

and

$$d\nu F(\nu) = \frac{d\nu}{\sqrt{2\pi}} \exp(-\frac{\nu^2}{2}) , \tag{10}$$

where $\rho$ is the mean mass density of the universe (background density) and $\nu_c = \frac{\delta_c}{\sigma_R}$. The number $\delta_c$ is the threshold for collapse, and we will use the value $\delta_c = 1.86$ which corresponds to neglecting the effects of dark energy.

According to linear perturbation theory, the density contrast $\sigma_R$ grows linearly in the scale factor before the contribution of dark energy to the equation of state of matter becomes important. The above formulas then show that for a Gaussian spectrum of fluctuations, the number density of nonlinear seeds falls off exponentially at high redshifts, thus making it difficult to account for the origin of the nonlinear seeds which are needed to explain the origin of SMBH. We will now investigate whether the addition of skewness and kurtosis can improve the prospects for high redshift SMBH formation.

## III. MODIFICATION OF THE HALO MASS FUNCTION BY PRIMORDIAL NON-GAUSSIANITIES

In this section we focus on the local type non-Gaussianities and study the modifications of the halo mass function when introducing primordial skewness and

kurtosis (see also [31, 32] for other studies of the effects of non-Gaussianities on structure formation). We take the primordial curvature fluctuations to be given by (1) in terms of a Gaussian distribution. Based on this relation the two, three and four point functions become

$$\begin{aligned}
\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle &= (2\pi)^3 P_\zeta(k_1)\delta^{(3)}(\vec{k}_1+\vec{k}_2) \quad (11)\\
\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle &= (2\pi)^3\frac{6}{5}f_{NL}(P_\zeta(k_1)P_\zeta(k_2)\\
&\quad +2\ perms.)\ \delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\\
\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\zeta(\vec{k}_4)\rangle &= (2\pi)^3\frac{54}{25}g_{NL}(P_\zeta(k_1)\\
&\quad P_\zeta(k_2)P_\zeta(k_3)+3\ perms.)\\
&\quad \delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\ .
\end{aligned}$$

In order to consider the effects of primordial non-Gaussianities on the smoothed density fluctuations, we can define the n-th central moment of the Probability Density Function (PDF) $F(\delta_R)d\delta_R$ in the following standard way

$$\langle\delta_R^n\rangle = \int_{-\infty}^{\infty}\delta_R^n F(\delta_R)d\delta_R\ . \qquad (12)$$

In addition, we can define the reduced $p$-th cumulant $S_p$ in this form

$$S_p(R) = \frac{\langle\delta_R^p\rangle_c}{\langle\delta_R^2\rangle_c^{p-1}}\ , \qquad (13)$$

where

$$\langle\delta_R\rangle_c = 0\ ,\ \langle\delta_R^2\rangle_c = \sigma_R^2\ ,\ etc. \qquad (14)$$

If we take into acount a non-Gaussian PDF of matter density perturbations in Eq. (12), we can build up the PDF from the cumulants and the Gaussian distribution by using the Edgeworth expansion. This technique gives the non-Gaussian PDF of the density field in terms of derivatives of the Gaussian PDF and reduced cumulants [31, 32]

$$F(\nu)d\nu = F_G(\nu)\ d\nu + \sum_{m=3}\frac{c_m}{m!}F_G^{(m)}(\nu)\ d\nu\ , \qquad (15)$$

with

$$\begin{aligned}
F_G(\nu) &= \frac{1}{\sqrt{2\pi}}exp(-\nu^2/2) \qquad (16)\\
F_G^{(m)}(\nu) &= \frac{d^m}{d\nu^m}F_G(\nu) = (-1)^m H_m(\nu)F_G(\nu)\ ,
\end{aligned}$$

where $F_G(\nu)$ is the Gaussian PDF and $H_m(\nu)$ are the Hermite polynomials

$$\begin{aligned}
H_1(\nu) &= \nu,\ H_2(\nu)=\nu^2-1,\ H_3(\nu)=\nu^3-3\nu\\
H_4(\nu) &= \nu^4-6\nu^2+3\ ,\ ...... \qquad (17)
\end{aligned}$$

The coefficients $c_m$ are given by

$$c_m = (-1)^m\int_{-\infty}^{\infty}H_m(\nu)F(\nu)d\nu\ , \qquad (18)$$

with

$$c_3 = -S_3(R)\sigma_R,\ ,\ c_4 = S_4(R)\sigma_R^2\ ,\ .... \qquad (19)$$

This technique allows us to calculate the non-Gaussian PDF of the density field in terms of the non zero values of the skewness and kurtosis of the primordial fluctuations. Inserting (16) into (15) and making use of (19) we obtain

$$\begin{aligned}
F(\nu)d\nu &= \frac{d\nu}{\sqrt{2\pi}}exp(-\nu^2/2)[1+\frac{S_3(R)\sigma_R}{6}H_3(\nu) \qquad (20)\\
&+\frac{1}{2}(\frac{S_3(R)\sigma_R}{6})^2H_6(\nu)+\frac{1}{6}(\frac{S_3(R)\sigma_R}{6})^3H_9(\nu)\\
&+\frac{S_4(R)\sigma_R^2}{24}H_4(\nu)+\frac{1}{2}(\frac{S_4(R)\sigma_R^2}{24})^2H_8(\nu)\\
&+\frac{1}{6}(\frac{S_4(R)\sigma_R^2}{24})^3H_{12}(\nu)+\ ...\ ]\ ,
\end{aligned}$$

where we are neglecting terms of higher order in $S_N(R)$ (from (11), (12) and (13)), and where

$$\begin{aligned}
S_3(R) &= \frac{6}{5}\frac{f_{NL}}{\sigma_R^4}\int\frac{dk_1}{k_1}W_R(k_1)\ \mathcal{A}(k_1)\mathcal{P}_\zeta(k_1) \qquad (21)\\
&\times\int\frac{dk_2}{k_2}W_R(k_2)\ \mathcal{A}(k_2)\mathcal{P}_\zeta(k_2)\\
&\times\int_{-1}^{1}\frac{d\mu_{12}}{2}W_R(k_{12})\ \mathcal{A}(k_{12})\left(1+\frac{P_\zeta(k_{12})}{P_\zeta(k_1)}+\frac{P_\zeta(k_{12})}{P_\zeta(k_2)}\right)\ ,
\end{aligned}$$

and

$$\begin{aligned}
S_4(R) &= \frac{54}{25}\frac{g_{NL}}{\sigma_R^6}\int\frac{dk_1}{k_1}W_R(k_1)\ \mathcal{A}(k_1)\mathcal{P}_\zeta(k_1) \qquad (22)\\
&\times\int\frac{dk_2}{k_2}W_R(k_2)\ \mathcal{A}(k_2)\mathcal{P}_\zeta(k_2)\\
&\times\int\frac{dk_3}{2\pi k_3}W_R(k_3)\ \mathcal{A}(k_3)\mathcal{P}_\zeta(k_3)\\
&\times\int_{-1}^{1}\frac{d\mu_{12}}{2}\int_{-1}^{1}\frac{d\mu_{123}}{2}\int_0^{2\pi}d\phi_{13}\ W_R(k_{123})\ \mathcal{A}(k_{123})\\
&\times\left(1+\frac{P_\zeta(k_{123})}{P_\zeta(k_1)}+\frac{P_\zeta(k_{123})}{P_\zeta(k_2)}+\frac{P_\zeta(k_{123})}{P_\zeta(k_3)}\right)\ ,
\end{aligned}$$

and where

$$\begin{aligned}
\mu_{ij} &= cos\ \theta_{ij}\ ,\\
k_{12} &= \sqrt{k_1^2+k_2^2+2k_1k_2\mu_{12}}\ \text{and} \qquad (23)\\
k_{123} &= \Big[k_1^2+k_2^2+k_3^2+2k_1k_2\mu_{12}+2k_1k_3\mu_{13}\\
&\quad +2k_2k_3\left(\sqrt{(1-\mu_{12}^2)(1-\mu_{13}^2)}\cos\phi_{13}+\mu_{12}\mu_{13}\right)\Big]^{1/2}\ .
\end{aligned}$$

Thus, the modified halo mass function based on the Press-Schechter formula with a non-Gaussian PDF of the smoothed density field is given by (from Eqs. (9) and

(20))

$$
\begin{aligned}
\frac{dn}{dM}\ (M,z)dM &= -dM\frac{2\rho}{M}\frac{d}{dM}\int_{\delta_c/\sigma_R}^{\infty} d\nu F(\nu) \qquad (24)\\
&= -dM\sqrt{\frac{2}{\pi}}\frac{\rho}{M}\exp[-\frac{\nu_c^2}{2}]\Bigg(\nu_c\frac{d\log\sigma_R}{dM}\Big[1\\
&+\frac{S_3\sigma_R}{6}H_3(\nu_c)+\frac{S_4\sigma_R^2}{24}H_4(\nu_c)\\
&+\frac{1}{2}H_6(\nu_c)\left(\frac{S_3\sigma_R}{6}\right)^2+\frac{1}{2}H_8(\nu_c)\left(\frac{S_4\sigma_R^2}{24}\right)^2\\
&+\frac{1}{6}H_9(\nu_c)\left(\frac{S_3\sigma_R}{6}\right)^3+\frac{1}{6}H_{12}(\nu_c)\left(\frac{S_4\sigma_R^2}{24}\right)^3\Big]\\
&+H_2(\nu_c)\frac{d}{dM}\left(\frac{S_3\sigma_R}{6}\right)+H_3(\nu_c)\frac{d}{dM}\left(\frac{S_4\sigma_R^2}{24}\right)\\
&+\frac{1}{2}H_5(\nu_c)\frac{d}{dM}\left(\frac{S_3\sigma_R}{6}\right)^2+\frac{1}{2}H_7(\nu_c)\frac{d}{dM}\left(\frac{S_4\sigma_R^2}{24}\right)^2\\
&+\frac{1}{6}H_8(\nu_c)\frac{d}{dM}\left(\frac{S_3\sigma_R}{6}\right)^3\\
&+\frac{1}{6}H_{11}(\nu_c)\frac{d}{dM}\left(\frac{S_4\sigma_R^2}{24}\right)^3\Bigg)+.....
\end{aligned}
$$

here $\nu_c = \frac{\delta_c}{\sigma_R}$ and $\delta_c$ is the threshold for collapse.

In what follows we consider two important forms of non-Gaussianity for cosmological observations, namely the local form and the equilateral form. The *local form* of the bispectrum requires that one of the three momentum modes exits the Hubble radius (e.g. in the context of an inflationary universe) much earlier than the other two, i.e $k_1 \ll k_2 \simeq k_3$. Taking this form of the non-Gaussianity for skewness and kurtosis we get

$$S_3(R) = \frac{12}{5}\frac{f_{NL}}{\sigma_R^2}\int\frac{dk_1}{k_1}W_R(k_1)\ \mathcal{A}(k_1)\mathcal{P}_\zeta(k_1)\ ,\qquad (25)$$

and

$$
\begin{aligned}
S_4(R) &= \frac{162}{25}\frac{g_{NL}}{\sigma_R^4}\int\frac{dk_1}{k_1}W_R(k_1)\ \mathcal{A}(k_1)\mathcal{P}_\zeta(k_1)\\
&\times\int\frac{dk_2}{k_2}W_R(k_2)\ \mathcal{A}(k_2)\mathcal{P}_\zeta(k_2)\ . \qquad (26)
\end{aligned}
$$

For the *equilateral* form of non-Gaussianity, i.e $k_1 = k_2 = k_3 = k_4 = k$, we have

$$S_3(R) = \frac{18}{5}\frac{f_{NL}}{\sigma_R^2}\int\frac{dk}{k}W_R(k)\ \mathcal{A}(k)\mathcal{P}_\zeta(k) \qquad (27)$$

and

$$S_4(R) = \frac{216}{25}\frac{g_{NL}}{\sigma_R^4}\Big(\int\frac{dk}{k}W_R(k)\ \mathcal{A}(k)\mathcal{P}_\zeta(k)\Big)^2\ . \qquad (28)$$

We can now estimate the magnitude of the skewness and kurtosis in the equilateral limit. Let us consider a sharp k-space filter which is particularly useful for theoretical arguments and is the equivalent to the top-hat filter in Fourier space [28],

$$W_R(k) = \begin{cases} 1 & kR \le 1\\ 0 & kR > 1\end{cases} \qquad (29)$$

In real space this takes the form

$$W_R(x) = \frac{3}{V}\ |\frac{x}{R}|^{-3}(\sin\frac{|x|}{R}-\frac{|x|}{R}\cos\frac{|x|}{R})\ , \qquad (30)$$

where the volume is taken to be $V = 6\pi^2R^3$. Therefore, we obtain

$$S_3(R)\sigma_R \simeq \frac{18}{5}f_{NL}\sqrt{\mathcal{P}_\zeta(1/R)} \simeq \frac{0.8}{5}\times 10^{-3}f_{NL} \qquad (31)$$

and

$$S_4(R)\sigma_R^2 \simeq \frac{216}{25}g_{NL}\mathcal{P}_\zeta(1/R) \simeq \frac{4.3}{25}\times 10^{-7}g_{NL}\ , \qquad (32)$$

where we used the approximations $n_s = 1$ and $A_s = 2.4\times 10^{-9}$. This analytical analysis shows that only primordial non Gaussianities with magnitudes $f_{NL} \geq 10^2$ and $g_{NL} \geq 10^6$ yield an important contribution to the modified halo mass function of Eq.(24).

Substituting Eq. (27) into the modified halo mass function Eq. (24) we can compute the mass of nonlinear halos with a mean separation $d$ as a function of redshift. Having such haloes is a necessary condition for the formation of super-massive black holes. Since there is evidence that every galaxy harbors a SMBH, we are interested in the value $d = d_{gal}$ corresponding to the mean comoving separation of current galaxies

$$d_{gal}^3\ M\ \frac{dn}{dM}(M,z) = 1\,. \qquad (33)$$

By using Eq. (33) and the CAMB code [33] we can plot the diagram of dark matter mass of halos with mean separation $d_{gal}$ as a function of redshift for different values of the non-Gaussianity parameters $f_{NL}$ and $g_{NL}$ and assuming equilateral shape. The results are shown in Figure 1 (see also [34] for an analysis of the halo mass function for Gaussian fluctuations).

Our results show that the effect of primordial non-Gaussianities given by $f_{NL}$ and $g_{NL}$ with values of these parameters consistent with the current observational bounds cannot change the number distribution of high redshift nonlinear dark matter halos in a way which will have an important impact on the formation of super-massive black holes at high redshifts. To have an important effect at a redshift $z = 20$ values of the non-Gaussianity parameters larger than $f_{NL} \geq 10^2$ or $g_{NL} \geq 10^6$ would be required.

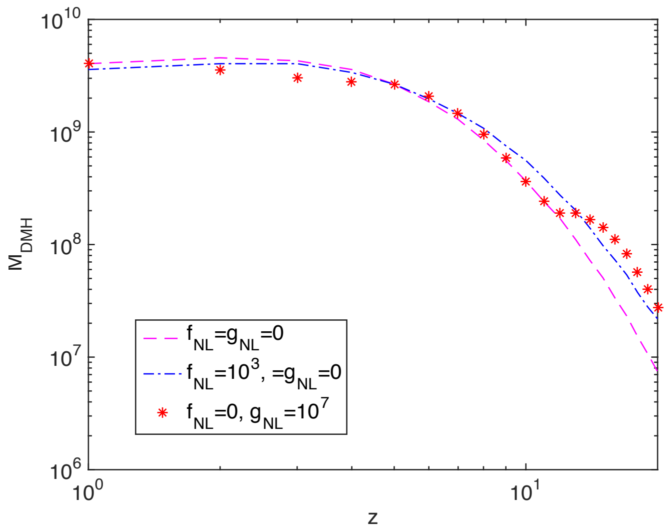


FIG. 1: Nonlinear dark matter halo mass (vertical axis, in solar mass units) as a function of redshift (horizontal axis, up to $z = 20$) obtained by using the CAMB [33] code for different values of the non-Gaussianity parameters $f_{NL}$ and $g_{NL}$ and assuming equilateral shape. The mass shown is the mass which is turning nonlinear on a co-moving length scales which corresponds to the separation $d_{gal} = 1\text{Mpc}$ of galaxies. We have made use of the value $\frac{t_0}{G} = 10^{23} M_{\odot}$. We have shown curves corresponding to the lowest values of $f_{NL}$ and $g_{NL}$ for which the non-Gaussianities have an important effect.

## IV. CONCLUSIONS

In the present paper we have studied whether introducing primordial non-Gaussianities in the density field in terms of skewness and kurtosis parameters $f_{NL}$ and $g_{NL}$ could have a large impact on the number density of nonlinear dark matter haloes at high redshifts. We found that values of $f_{NL}$ and $g_{NL}$ much larger than the current observational bounds are required in order to change the predictions appreciably. Hence, density distributions which are Gaussian modulo skewness and kutosis parameters $f_{NL}$ and $g_{NL}$ cannot have a big impact on the formation of high redshift super-massive black holes. This result contrasts with the large effect which cosmic string loops can have [16] at high redshift. In the case of cosmic strings, the distribution is intrinsically non-Gaussian and not well described at all by a Gaussian process plus skewness and kurtosis.

## V. ACKNOWLEDGMENTS

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